Plant Geometry from Fractals
Based on L-systems Theory

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Objective

• To generate plant geometries that place leaves in locations similar to the placement in chamise and Utah juniper
Introduction to Bush Model

- Used for broadleaf species
- Leaves randomly placed in specified geometry (sphere, parabola, rectangle, etc.)
Rewriting: from Simple to Complex

Koch Snowflake

- Replaces each line of the initiator with the generator
- Increasingly complicated
- Self-similar design

DOL-systems

- Simplest class of L-systems
- Rewrites strings instead of shapes according to a specific set of rules
- Each time of rewriting is called a derivation

Initiator

Generator
Turtle Interpretation: Visually Representing L-systems Strings

- Creates shape according to string with the following symbols:
  
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>forward one ‘step’ of length $d$</td>
</tr>
<tr>
<td>+</td>
<td>turn right in $x$-plane by angle $\delta$</td>
</tr>
<tr>
<td>−</td>
<td>turn left in $x$-plane by angle $\delta$</td>
</tr>
<tr>
<td>*</td>
<td>turn right in $y$-plane by angle $\delta$</td>
</tr>
<tr>
<td>!</td>
<td>turn left in $y$-plane by angle $\delta$</td>
</tr>
<tr>
<td>X</td>
<td>initial string to be replaced</td>
</tr>
</tbody>
</table>

- Each symbol in string consecutively carried out
- X treated as F for last derivation to end rewriting
- Symbols to change angle in $y$-plane added to original L-systems symbols for three-dimensions
Example of Turtle Interpretation

Rewriting rule: replace ‘X’ with ‘F +X –X F X’

Before Rewriting

\[ X \]

\( n = 2 \)

\[ F + F + X -X F X - F + X -X F X F F +X -X F X \]

\( n = 1 \)

\[ F +X -X F X \]

\( n = 3 \)

\[ F + F + F +X -X F X - F +X -X F X F F +X -X F \]

\[ X - F + F +X -X F X - F +X -X F X F F +X -X F \]

\[ X F F + F +X -X F X - F +X -X F X F F +X -X F X \]

\[ X \]
L-systems in 2-D

• Every F in string is one step forward of length, \(d\), and angle, \(\delta\)

\[ X = F + X F + X - X \]

\(n = 7\)

\(\delta = 20^\circ\)

\(\epsilon = 0.5\)

(polar coordinates)

• Number of derivations specified by the variable, \(n\)

• Length of next derivation determined by a scaling factor, \(\epsilon\):
Applying L-systems to 3-D

- Every $F$ steps forward in spherical coordinates

$$X = F - !X \ F + !X \ast X$$

- Each $+/-$ changes $\alpha$ in $x$-plane and $\ast/!$ changes $\beta$ in $y$-plane
- Spherical angles, $\varphi$ and $\theta$, calculated from 2-d angles, $\alpha$ and $\beta$, before each $F$
Stochastic L-systems

- Equal probability of choosing one of five strings
- Each derivation uses a randomly chosen string
Strings for Chamise and Juniper

- Studied smallest branch segment

- Used measured average and standard deviation to decide number of ‘F’\textquotesingle s and ‘X’\textquotesingle s in strings
Strings for Each Species

Chamise
\[
\begin{align*}
X_1 &= F -X F +X *X F +!X \\
X_2 &= F +*X F +!X F +*X F +!X \\
X_3 &= F +!X -*X F +!X *X F -X \\
X_4 &= F +*X F -*X -*X +X +!X !X \\
X_5 &= F -*X F *X +*X F -*X 
\end{align*}
\]

Utah juniper
\[
\begin{align*}
X_1 &= F +*X F +X -X !X \\
X_2 &= F -*X F +*X F -X !X \\
X_3 &= F +!X F -*X F +X F +*X -*X !X \\
X_4 &= F -!X F +*X -*X F -X F +X F ++X !X \\
X_5 &= F +*X F -*X F +X F +!X F -X F +*X -*X X 
\end{align*}
\]
Modeling a Chamise Shrub using L-systems

- Equally divides 3-dimensional space between the number of branches
- Can specify an average distance from the origin so all branches do not come out of one central point
## Chamise Measurements

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
<th>*based on measurements made in lab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average primary branch angle (with 0° being vertical)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle between secondary and primary branches</td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>Length of secondary / length of primary</td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>Average distance of primary branches from the center of the shrub</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Number of Branches for Chamise

• Number of branches determined by crown diameter
• Fuel split into four fuel classes

<table>
<thead>
<tr>
<th>Crown Diameter</th>
<th># Branches</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 cm</td>
<td>98</td>
</tr>
<tr>
<td>350 cm</td>
<td>117</td>
</tr>
<tr>
<td>400 cm</td>
<td>139</td>
</tr>
<tr>
<td>450 cm</td>
<td>166</td>
</tr>
</tbody>
</table>
Comparison of Chamise Geometry
• Bulk density of generated geometry falls within range of naturally-occurring species (0.5-2.8 kg/m³)
Modeling a Utah Juniper Shrub using L-systems

- Primary branches distributed along bush height
- Secondary branches determined by L-systems string
- Fuel spaced along secondary branches by certain distance
- Primary branches bent ("curved")
<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum primary branch angle</td>
<td></td>
</tr>
<tr>
<td>Secondary branch angle</td>
<td></td>
</tr>
<tr>
<td>Length of secondary / length of primary</td>
<td></td>
</tr>
<tr>
<td>Length of highest primary branch / length of lowest primary branch</td>
<td></td>
</tr>
<tr>
<td>Angle of highest primary branch / angle of lowest primary branch</td>
<td></td>
</tr>
<tr>
<td>Distance between highest two primary branches / distance between lowest primary branches</td>
<td></td>
</tr>
<tr>
<td>Distance between fuel elements</td>
<td></td>
</tr>
<tr>
<td>Angle between fuel and primary/secondary branch</td>
<td></td>
</tr>
</tbody>
</table>
Number of Branches for Juniper

- Data from Mason, L.R., Hutchings, S.S., 1967
- Three fuel denseness classes – sparse, medium, or dense
- Number of branches determined by crown density

- **Sparse**
  - 76 branches

- **Medium**
  - 106 branches

- **Dense**
  - 152 branches
Number of Branches for Juniper

• Additional correlation made using data collected
• Predicts the number of branches to give the correct fuel weight for the specified crown diameter

• Measured bulk density higher than data from literature
Comparison of Utah Juniper Geometry
Comparison of Utah Juniper Geometry

Shrub Measurements
Height: 140 cm
Crown Diameter: 84 cm
Dry Mass: 663.9 g

Model Measurements
Height: 140 cm
Crown Diameter: 78 cm
Dry Mass: 855.6 g
Close-up Comparison
Utah Juniper
Dry Mass
Conclusions

• L-Systems used to portray shrub geometry based on fuel elements
• Chamise geometry has bulk density within range
• Juniper geometry has dry mass within range of naturally-occurring species
Shrub Combustion Model for Broadleaf Species

- Leaves randomly placed in specified shape (e.g. box, sphere, hollow box or sphere, parabolic)